A CLP Approach to Detection and Identification of Control System Component Failures*

Yousri El Fattah
Christian Holzbaur

Austrian Research Institute for Artificial Intelligence
Schottengasse 3, A-1010 Wien, Austria
Phone: +43-1-5336112
Fax: +43-1-5320652
Email: {yousri,christian}@ai.univie.ac.at

June 24, 1994

Abstract

One approach to fault detection and identification (FDI) in control systems is based on computing the so-called parity relations for sensors and actuators. We state the problem of generating those parity relations in the language of constraints, and describe an implementation of a parity solver in the Constraint Programming Language CLP($\Re$). The solver adopts a discrete linear time-invariant (LTI) model of control systems, and outputs a set of single-component parity relations. We describe a FDI procedure, also in CLP($\Re$), that monitors system behavior and computes single-fault diagnoses. The CLP approach enhances the naturalness of representation of system relations, and makes use of the resolution capability of CLP both for deriving parity relations and for making diagnostic decisions. An example is given to illustrate the viability of the CLP approach.

*This work was supported by the Austrian "Fonds zur Förderung der wissenschaftlichen Forschung" under grant P9426-PHY. Austrian Research Institute for Artificial Intelligence is supported by the Austrian Federal Ministry of Science and Research.

1 Introduction

Traditionally, fault tolerance in complex plants and their control systems is achieved through the use of physical redundancy. More recently, interest has shifted to the use of model-based approaches, which rely instead on the use of *analytical redundancy*. Analytical redundancy depends on a model of the dynamics of the controlled system to generate *residual* function of the temporal history of the actuator commands and sensor outputs which has the property of being small when the system is operating normally. Examples of this approach are the observer-based method [2, 8] and the parity space method [1, 5, 9]. Both methods are different ways for solving the same kind of problem, and their equivalence has been shown [4].

Approaches to model-based diagnosis of dynamic systems have also been developed in artificial intelligence. Those approaches mainly adopt logic-based formulation and consider qualitative system models [7]. The motivation for our work is to establish a connection between the model-based diagnosis approaches in engineering and in artificial intelligence. Specifically, we provide a constraint-based formulation of the problem of generating parity relations for dynamic control systems. We provide for the first time a clear, general interpretation of a parity relation as a projection operation of a set of linear equations. The advantage of this interpretation is that it allows a direct mapping into the language of constraint logic programming CLP($\Re$) [3]. This proposed CLP framework enhances the naturalness of representation of system relations, and makes use of the resolution capability of CLP both for deriving parity relations and for making diagnostic decisions.

The paper is structured as follows. Section 2 gives a CLP approach to failure detection and identification. Section 3 describes a simulated example to illustrate the viability of the proposed approach. Section 4 gives concluding remarks.

2 Failure Detection and Identification

In this section we describe a CLP approach for failure detection and identification (FDI) of control system component failures. We consider discrete linear time-invariant (LTI) multi-input multi-output (MIMO) systems, described by the predicates:

$$one\_transition(System, t, u(t), x(t), x(t+1)) \quad (1)$$

$$obs(System, t, x(t), u(t), y(t)) \quad (2)$$

where t denotes the time epoch $(0, 1, 2, \ldots)$, $x(t) \in R^n, u(t) \in R^m, y(t) \in R^l$ denote respectively the state, control, and output tuples (vectors) at epoch t. The dimension n of the state vector is the *system order*. Each i-th component $(1 \leq i \leq m)$ of the input u is associated with a distinct *actuator*, and each j-th component $(1 \leq j \leq l)$ of the output y is associated with a distinct *sensor*. The predicate *one_transition* specifies the constraint relation between the state and control tuples at any given epoch t and the state tuple at the consecutive epoch $t+1$. The predicate *obs* specifies the constraint between the state, control, and observation tuples at any given epoch t. For LTI systems, the *one_transition* and *obs* relations are both linear constraints, and can be expressed in matrix notation as follows,

$$\begin{aligned} x(T+1) &= A\ x(T) + B\ u(T) && (3) \\ y(T) &= C\ x(T) + D\ u(T) && (4) \end{aligned}$$

where A, B, C, D are constant matrices of proper dimensions.

Let the current epoch be t and consider the system behavior over the past k epochs. Let $U(t,k), Y(t,k)$ be the input and output sequences over epochs between t and $t-k$,

$$\begin{aligned} U(t,k) &= (u(t), u(t-1), \ldots, u(t-k)) && (5) \\ Y(t,k) &= (y(t), y(t-1), \ldots, y(t-k)) && (6) \end{aligned}$$

Based on eq. 3, it can be shown that the current state at t can be expressed as a function of the state at $t-k$ and the input sequence of length $k-1$ between $t-k$ and $t-1$ ($k \geq 1$),

$$x(t) = A^k\ x(t-k) + [B]_k\ U(t-1, k-1) \tag{7}$$

where,

$$[B]_k = \begin{pmatrix} B & AB & A^2B & \ldots & A^{k-1}B \\ 0 & B & AB & \ldots & A^{k-2}B \\ 0 & 0 & B & \ldots & A^{k-3}B \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & B \end{pmatrix} \tag{8}$$

Let $ok\_sequence(t, k, U(t,k), x(t-k), x(t), Y(t,k))$ be a predicate characterizing the relation between the states at t, $t-k$, the input sequence, $U(t,k)$ and the output sequence, $Y(t,k)$, such that all variables are consistent with the system's normal behavior according to eqs. 3, 4. *ok_sequence* can be defined recursively as follows,

$$ok\_sequence(t, 0, u(t), x(t), x(t), y(t)) \leftarrow obs(t, x(t), u(t), y(t)). \tag{9}$$

$$\begin{aligned} & ok\_sequence(t, k, U(t,k), x(t-k), x(t), Y(t,k)) \leftarrow \\ & \quad ok\_sequence(t-1, k-1, U(t-1,k-1), x(t-k), x(t-1), Y(t-1,k-1)) \wedge \\ & \quad one\_transition(t-1, u(t-1), x(t-1), x(t)) \wedge \\ & \quad obs(t, x(t), u(t), y(t)) \wedge \\ & \quad U(t,k) = U(t-1,k-1) \cup u(t) \wedge \\ & \quad Y(t,k) = Y(t-1,k-1) \cup y(t). \end{aligned} \tag{10}$$

ok_sequence yields the constraint relations given by eq. 7, as well as the following,

$$Y(t,k) = [C]_k\ x(t-k) + [D]_k\ U(t,k) \tag{11}$$

Table 1: **Failure Detection and Identification**

Input: LTI system model and its observed input-output sequence. **Output:** A diagnostic pair (D_s, D_a), consisting of sets of culprit sensors and actuators. **Initialize:** $t \leftarrow 0$, $u(-1,n) \leftarrow \emptyset$, $y(-1,n) \leftarrow \emptyset$. 1. Compute parity, $sspr(i, U(t,n), Y(T,n))$ for each sensor i. 2. Compute parity, $sapr(j, U(t,n), Y(T,n))$ for each actuator j. 3. $u(t,n) = u(t-1, n-1) \cup u(t)$, $y(t,n) = y(t-1, n-1) \cup y(t)$. 4. $S = \{i \mid sspr(i, u(t,n), y(t,n)) \ is \ inconsistent\}$, $A = \{j \mid sapr(j, u(t,n), y(t,n)) \ is \ inconsistent\}$ 5. Output $(\emptyset, S)$ if $\mid A \mid \geq \mid S \mid$; $(A, \emptyset)$ if $\mid S \mid > \mid A \mid$. 6. $t \leftarrow t+1$. 7. Go to 3.

where,

$$[C]_k = \begin{pmatrix} CA^k \\ CA^{k-1} \\ CA^{k-2} \\ \vdots \\ CA \\ C \end{pmatrix} \quad [D]_k = \begin{pmatrix} D & CB & CAB & CA^2B & \dots & CA^{k-1}B \\ 0 & D & CB & CAB & \dots & CA^{k-2}B \\ 0 & 0 & D & CB & \dots & CA^{k-3}B \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & D & CB \\ 0 & 0 & 0 & \dots & 0 & D \end{pmatrix} \tag{12}$$

The well-known Cayley-Hamilton theorem [6] implies that increasing k beyond the system order n does not contribute any additional constraint on the system variables. Thus, $ok\_sequence(t, n, U(t,n), x(t-n), x(n), Y(t,n))$ sufficiently characterizes the system's normal behavior. Parity relations are those relations between input and output sequences that are valid for all system states. Parity relations are obtained from $ok\_sequence(t, n, U(t,n), x(t-n), x(n), Y(t,n))$ by eliminating the state x, or projecting $ok\_sequence$ on the (U, Y) sequence. Projection is a part of the resolution strategy of CLP($\Re$). Parity relations can be specialized to single-component, i.e. single-sensor (SSPR) or single-actuator (SAPR). SSPR for sensor i, denoted as $sspr(i, U(t,n), Y(t,n))$, is a set of constraint equations on the values of the sequence pair: $(U(t,n), Y^{(i)}(t,n))$, where $Y^{(i)}(t,n)$ is the specialization of the output sequence to the i-th sensor. SAPR for actuator j, denoted as $sapr(j, U(t,n), Y(t,n))$, is a set of constraint equations on the values of the sequence pair: $(U^{(j)}(t,n), Y(t,n))$, where $U^{(j)}(t,n)$ is the specialization of the input sequence to the j-th

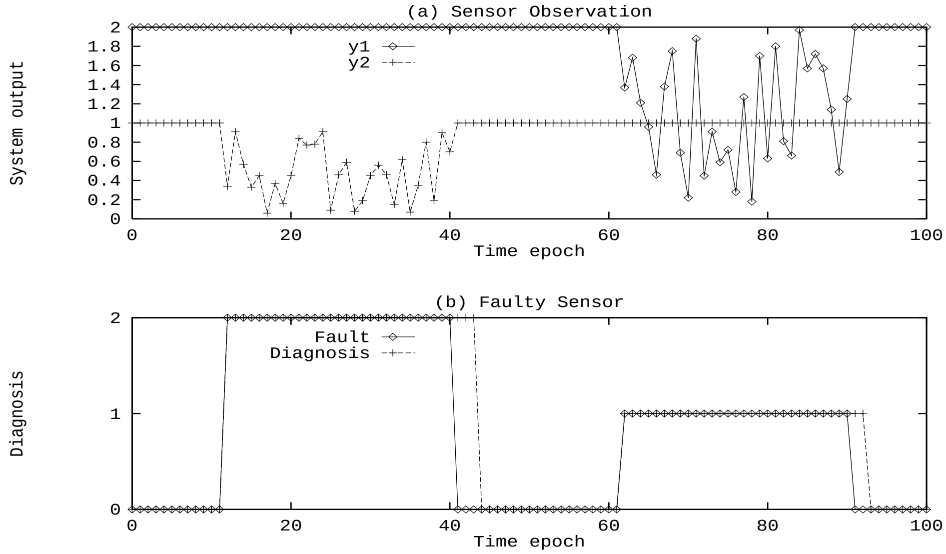

Figure 1: (a) Output with sensor fault, (b) Diagnosis vesrus actual fault

actuator.

Table 1 states a CLP-based FDI procedure, which computes single component failures accounting for the input output sequence. It does that in two steps. First, the single-component parity relations are computed for both sensors and actuators. Second, a decision is made on the basis of consistency of those relations with actual input-output observation. Diagnosis consists of a pair of sets of sensors and actuators whose members are single fault candidates. The logic in the failure decision step 5 prefers a failure explanation that includes least number of candidates. For instance, upon the failure of a sensor, just one of the sensor parity relations and all, or at least most, of the actuator relations would become inconsistent. The FDI procedure (table 1) would decide that one sensor had failed rather than that most of the actuators had simultaneously failed. (See [5].)

3 Application

In this section we describe the results of applying our CLP-based approach to a small example [5]. The control system model is given by,

```
one_transition(3,T,[u1(T):U1,u2(T):U2],[x1(T):_,x2(T):X2,x3(T):X3],
  [x1(T+1):X1p,x2(T+1):X2p,x3(T+1):X3p]) :-
        X1p = U1,
        X2p = 2*X2 + U2,
        X3p = X3 + U1 + U2.
obs(3,T,[u1(T):_,u2(T):_],[x1(T):X1,x2(T):X2,x3(T):X3],[y1(T):X1+X2,y2(T):X3]).
```

The SSPR relation generated by CLP($\Re$) for sensor 1 consists of the input-output sequence pair,

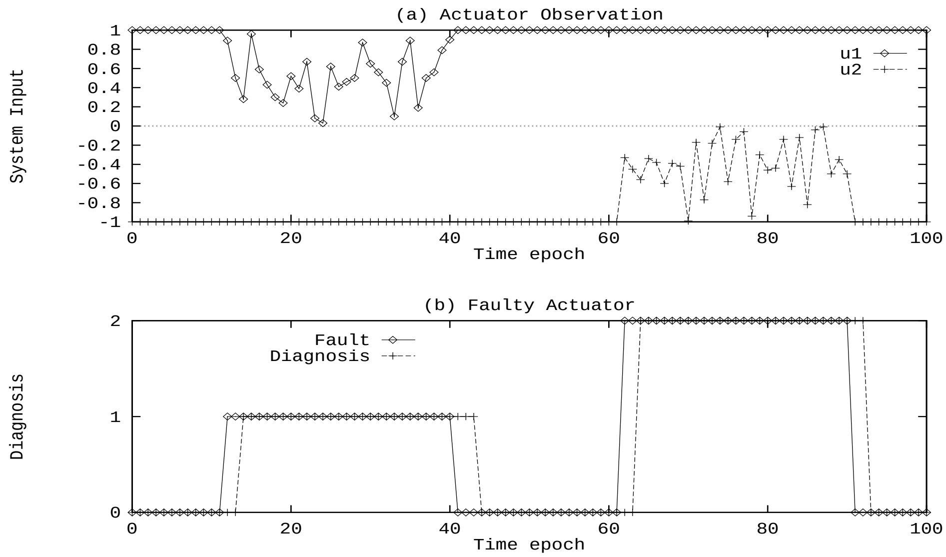

Figure 2: (a) Input with actuator fault, (b) Diagnosis versus actual fault

```
[[u1(_E):_I,u2(_E):_J],[u1(-1+_E):_B,u2(-1+_E):_A],
 [u1(-2+_E):_C,u2(-2+_E):_F],[u1(-3+_E):_G,u2(-3+_E):_K]],
[y1(_E):2.0*_D-2.0*_C+_B+_A,y1(-1+_E):_D,
 y1(-2+_E):0.5*_D+_G-0.5*_C-0.5*_F,y1(-3+_E):_H]
```

which is equivalent to the difference equation [5],

$$-2u_1(t-2) + u_1(t-1) + u_2(t-1) - y_1(t) + 2y_1(t-1) = 0$$

The SAPR relation generated by CLP($\Re$) for actuator 1 is as follows,

```
[u1(_E):_J,u1(-1+_E):_K,u1(-2+_E):_C,u1(-3+_E):_G],
[[y1(_E):2.0*_D-2.0*_C-_B+_A,y2(_E):_A],[y1(-1+_E):_D,y2(-1+_E):_B],
 [y1(-2+_E):0.5*_D+_G+0.5*_F-0.5*_B,y2(-2+_E):_F],[y1(-3+_E):_H,y2(-3+_E):_I]]
```

which is equivalent to the difference equation [5],

$$u_1(t-2) + 0.5y_1(t) - y_1(t-1) - 0.5y_2(t) + 0.5y_2(t-1) = 0$$

The parity relations generated for the second sensor and actuator (not shown) are also equivalent to those given in [5], based on transfer matrix techniques.

We simulated the control system with the initial state, ($x_1(0) = 1, x_2(0) = 1, x_3(0) = 1$) and constant control ($u_1(t) = 1, u_2(t) = -1$). A fault in an actuator or a sensor is considered to be a multiplicative random gain between zero and one. Fig. 1(a) shows the observed system output where the second sensor fails throughout the time interval (10,40), and the first sensor fails throughout the interval (60,90). Fig. 1(b) shows the diagnosis and the actual fault. The figure shows a good monitoring capability; diagnosis instantly tracks the fault at the time the fault begins. As the fault ends, the normal diagnosis is also restored albeit with some delay.

The delay is due to the dependence of the parity relations not only on current observation but also on previous ones. Similar performance is obtained for faulty actuators. Fig. 2(a) shows the observed system input where the first actuator fails throughout the time interval (10,40), and the second actuator fails throughout the interval (60,90). Fig. 2(b) shows the diagnosis and the actual fault. Unlike sensor fault, actuator faults are not detected instantly at the time they occur. The detection delay for actuator fault is caused by the time lag in their parity residual function.

4 Concluding Remarks

We formulate the construction of parity relations for dynamic control systems in the language of constraints. Parity relations are considered to be of a form suitable for single-component diagnosis of sensors or actuators. We give for the first time a new interpretation of a parity relation as a projection operation of a set of linear equations. The advantage of that interpretation is that it allows a direct mapping into the language of constraint logic programming CLP($\Re$). This proposed CLP framework enhances the naturalness of representation of system relations, and makes use of the resolution capability of CLP for both deriving parity relations and for making diagnostic decisions. Future work will extend the current approach by addressing the problem of robustness, namely the design of FDI methods which minimize sensitivity of diagnostic performance to model errors and uncertainties.

Acknowledgement

Thanks to Igor Mozetič for helpful comments on this paper.

References

[1] E.Y. Chow and A.S. Willsky. Analytical redundancy and the design of robust failure detection systems. *IEEE Trans. Automatic Control*, AC-29:603–615, 1984.

[2] R.N. Clark, D.C. Fosth, and V.M. Walton. Detecting instrument malfunctions in control systems. *IEEE Trans. Aerosp. Electron. Syst.*, AES-11(4):465–473, 1975.

[3] Y. El Fattah. Constraint logic programming: Applications and implications. *Artificial Intelligence in Engineering*, 7(3):175–182, 1993.

[4] P.M. Frank. Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy–a survey. *Automatica*, 26:459–474, 1990.

[5] M.A. Massoumnia and W.E.V. Velde. Generating parity relations for detecting and identifying control system component failures. *J. Guidance*, 11:60–65, 1988.

[6] R.H. Middleton and G.C. Goodwin. *Digital Control and Estimation: A Unified Approach.* Prentice-Hall,Inc., Engglewood Cliffs, NJ, 1990.

[7] H. T. Ng. Model-based, multiple fault diagnosis of time-varying, continuous physical devices. In *Proceedings of the Sixth Conference on Artificial Intelligence Applications*, pages 9–15, Los Alamitos, CA, 1990. IEEE Computer Society Press.

[8] R.J. Patton and S.M. Kangethe. Robust fault diagnosis using eigenstructure assignment of observers. In R.Patton, P. Frank, and R. Clark, editors, *Fault Diagnosis in Dynmaic Systems: Theory and Application*, pages 99–154. Prentice-Hall, Englewood Cliffs, NJ, 1989.

[9] J. Ragot and D. Maquin. An algorithm for obtaining the redundancy equations of LTI systems. *Automatica*, 30(3):537–542, 1994.